Reliability Prediction Basics

Reliability predictions are one of the most common forms of reliability analysis. Reliability predictions predict the failure rate of components and overall system reliability. These predictions are used to evaluate design feasibility, compare design alternatives, identify potential failure areas, trade-off system design factors, and track reliability improvement.

The Role of Reliability Prediction

Reliability Prediction has many roles in the reliability engineering process. The impact of proposed design changes on reliability is determined by comparing the reliability predictions of the existing and proposed designs.

The ability of the design to maintain an acceptable reliability level under environmental extremes can be assessed through reliability predictions. Predictions can be used to evaluate the need for environmental control systems.

The effects of complexity on the probability of mission success can be evaluated by performing a reliability prediction analysis. Results from the analysis may determine a need for redundant systems, back-up systems, subsystems, assemblies, or component parts**.** **MIL-HDBK-217** (Electronics Reliability Prediction), **Bellcore/Telcordia** (Electronics Reliability Prediction) and **NSWC** (Mechanical Reliability Prediction) provide failure rate and MTBF (Mean Time Between Failures) data for electronic and mechanical parts and equipment.

A reliability prediction can also assist in evaluating the significance of reported failures. Ultimately, the results obtained by performing a reliability prediction analysis can be useful when conducting further analyses such as a **FMECA** (**Failure Modes, Effects and Criticality Analysis**), **RBD** (**Reliability Block Diagram**) or a **Fault Tree** analysis. The reliability predictions are used to evaluate the probabilities of failure events described in these alternate failure analysis models.

Reliability and Unreliability

First, let us review some concepts of reliability. At a given point in time, a component or system is either functioning or it has failed, and that the component or system operating state changes as time evolves. A working component or system will eventually fail. The failed state will continue forever, if the component or system is non-repairable. A repairable component or system will remain in the failed state for a period of time while it is being repaired and then transcends back to the functioning state when the repair is completed. This transition is assumed to be instantaneous. The change from a functioning to a failed state is **failure** while the change from a failure to a functioning state is referred to as **repair**. It is also assumed that repairs bring the component or system back to an “as good as new” condition. This cycle continues with the repair-to-failure and the failure-to-repair process; and then, repeats over and over for a repairable system.

The reliability prediction standards such as MIL-217, Bellcore/Telcordia and NSWC Mechanical assume the component or system to be non-repairable, in a new condition at time zero and have a constant failure rate, if evaluated over a very long time period and using an infinite or very large sample size of components or systems.

Reliability (for non-repairable items) can be defined as the probability that an item will perform a defined function without failure under stated conditions for a stated period of time. One must grasp the concept of probabilities in order to understand the concept of reliability. The numerical values of both reliability and unreliability are expressed as a probability from 0 to 1 and have no units.

Reliability stated in another way:

The **Reliability**, ***R(t)***, of a component or system is defined as the probability that the component or system remains operating from time zero to time t_1, given that it was operating at time zero.

Or stated another way for repairable items:

The **Reliability**, ***R(t)***, is defined as the probability that the component or system experiences no failures during the time interval zero to t_1 given that the component or system was repaired to a like new condition or was functioning at t_0.

And:

The **Unreliability**, ***F(t)***, of a component or system is defined as the probability that the component or system experiences the first failure or has failed one or more times during the time interval zero to time t, given that it was operating or repaired to a like new condition at time zero.

Or stated another way:

The **Unreliability**, ***F(t)***, of a component or system at a given time is simply the number of components failed to time *t* divided by the total number of samples tested.

The following relationship holds true since a component or system must either experience its first failure in the time interval zero to t or remain operating over this period.

$$R(t) + F(t) = 1 \qquad \text{or} \qquad \textit{Unreliability}\ F(t) = 1 - R(t)$$

Availability and Unavailability

In reliability engineering and reliability studies, it is the general convention to deal with unreliability and unavailability values rather than reliability and availability. The numerical value of both availability and unavailability are also expressed as a probability from 0 to 1 with no units.

The **Availability**, ***A(t)***, of a component or system is defined as the probability that the component or system is operating at time *t*, given that it was operating at time zero.

The **Unavailability**, ***Q(t)***, of a component or system is defined as the probability that the component or system is not operating at time *t*, given that is was operating at time zero.

Or stated another way:

Unavailability, *Q(t)* is the probability that the component or system is in the failed state at time *t* and is equal to the number of the failed components at time t divided by the total sample.

Therefore, the following relationship holds true since a component or system must be either operating or not operating at any time:

$$A(t) + Q(t) = 1$$

Both parameters are used in reliability assessments, safety and cost related studies. The following relationship holds:

$$\text{Unavailability } Q(t) \leq \text{Unreliability } F(t)$$

For a non-repairable component or system:

$$\text{Unavailability } Q(t) = \text{Unreliability } F(t)$$

NOTE: This general equality only holds for system unavailability and unreliability if all the components within the system are non-repairable up to time *t*.

Reliability Prediction Definitions

Failure Rates

Reliability predictions are based on failure rates.

Conditional Failure Rate or Failure Intensity, $\lambda(t)$, can be defined as the anticipated number of times an item will fail in a specified time period, given that it was as good as new at time zero and is functioning at time *t*.

It is a calculated value that provides a measure of reliability for a product. This value is normally expressed as failures per million hours (fpmh or 10^6 hours), but can also be expressed, as is the case with Bellcore/Telcordia, as failures per billion hours (fits or failures in time or 10^9 hours). For example, a component with a failure rate of 2 failures per million hours would be expected to fail 2 times in a million-hour time period.

Failure rate calculations are based on complex models which include factors using specific component data such as temperature, environment, and stress. In the prediction model, assembled components are structured serially. Thus, calculated failure rates for assemblies are a sum of the individual failure rates for components within the assembly.

There are three common basic categories of failure rates:

Mean Time Between Failures (MTBF)

Mean Time To Failure (MTTF)

Mean Time To Repair (MTTR)

Mean Time Between Failures (MTBF)

Mean time between failures (MTBF) is a basic measure of reliability for repairable items. MTBF can be described as the time passed before a component, assembly, or system fails, under the condition of a constant failure rate. Another way of stating MTBF is the expected value of time between two consecutive failures, for repairable systems. It is a commonly used variable in reliability and maintainability analyses.

MTBF can be calculated as the inverse of the failure rate, λ, for constant failure rate systems. For example, for a component with a failure rate of 2 failures per million hours, the MTBF would be the inverse of that failure rate, λ, or:

$$MTBF = \frac{1}{\lambda} \quad OR \quad \frac{1}{2\ Failures/10^6\ hours} = 500{,}000\ hours\ /\ failure$$

> **NOTE:** Although MTBF was designed for use with repairable items, it is commonly used for both repairable and non-repairable items. For non-repairable items, MTBF is the time until the first (an only) failure after t_0.

Mean Time To Failure (MTTF)

Mean time to failure (MTTF) is a basic measure of reliability for non-repairable systems. It is the mean time expected until the first failure of a piece of equipment. MTTF is a statistical value and is intended to be the mean over a long period of time and with a large number of units. For constant failure rate systems, MTTF is the inverse of the failure rate, λ. If failure rate, λ, is in failures/million hours, MTTF = 1,000,000 / Failure Rate, λ, for components with exponential distributions. Or

$$MTTF = \frac{1}{\lambda\ failures/10^6\ hours}$$

For repairable systems, MTTF is the expected span of time from repair to the first or next failure.

Mean Time to Repair (MTTR)

Mean time to repair (MTTR) is defined as the total amount of time spent performing all corrective or preventative maintenance repairs divided by the total number of those repairs. It is the expected span of time from a failure (or shut down) to the repair or maintenance completion. This term is typically only used with repairable systems.

Failure Frequencies

There are four failure frequencies, which are commonly used in reliability analyses.

- **Failure Density** $f(t)$ - The failure density of a component or system, $f(t)$, is defined as the probability per unit time that the component or system experiences its first failure at time *t*, given that the component or system was operating at time zero.
- **Failure Rate** $r(t)$ - The failure rate of a component or system, $r(t)$, is defined as the probability per unit time that the component or system experiences a failure at time *t*, given that the component or system was operating at time zero and has survived to time *t*.
- **Conditional Failure Intensity (or Conditional Failure Rate)** $\lambda(t)$ - The conditional failure intensity of a component or system, $\lambda(t)$, is defined as the probability per unit time that the component or system experiences a failure at time *t*, given that the component or system was operating, or was repaired to be as good as new, at time zero and is operating at time *t*.
- **Unconditional Failure Intensity or Failure Frequency** $\omega(t)$ - The unconditional failure intensity of a component or system, $\omega(t)$, is defined as the probability per unit time that the component or system experiences a failure at time *t*, given that the component or system was operating at time zero.

Relationships Between Failure Parameters

The following relations exist between failure parameters.

$$R(t) + F(t) = 1$$

$$f(t) = \frac{dF(t)}{dt}$$

$$F(t) = \int_0^t f(u).du$$

$$r(t) = \frac{f(t)}{1 - F(t)}$$

$$R(t) = e^{-\int_0^t r(u).du}$$

$$F(t) = 1 - e^{-\int_0^t r(u).du}$$

$$f(t) = r(t)e^{-\int_0^t r(u).du}$$

The definitions for failure rate $r(t)$ and conditional failure intensity $\lambda(t)$ differ in that that the failure rate definition addresses the first failure of the component or system rather than any failure of the component or system. In the special cases of the failure rate being constant with respect to time or if the component is non-repairable these two quantities are equal. In summary :

$\lambda(t) = r(t)$ for non-repairable components

$\lambda(t) = r(t)$ for constant failure rates

$\lambda(t) \neq r(t)$ for the general case

The difference between the conditional failure intensity (CFI) $\lambda(t)$ and unconditional failure intensity $\omega(t)$ is that the CFI has an additional condition that the component or system has survived to time *t*. The relationship between these two quantities may be expressed mathematically as

$$\omega(t) = \lambda(t)[1 - Q(t)]$$

For most reliability and availability studies the unavailability $Q(t)$ of components and systems is very much less than 1. In such cases

$$\omega(t) \approx \lambda(t)$$

Constant Failure Rates

If the failure rate is constant then the following expressions apply :

$$R(t) = e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$f(t) = \lambda e^{-\lambda t}$$

As can be seen from the equation above a constant failure rate results in an exponential failure density distribution.

Repairable and Non-repairable Items

It is important to distinguish between repairable and non-repairable items when predicting or measuring reliability.

Non-repairable items

Non-repairable items are components or systems such as a light bulb, transistor, rocket motor, etc. Their reliability is the survival probability over the items expected life or over a specific period of time during its life, when only one failure can occur.
During the component or systems life, the instantaneous probability of the first and only failure is called the **hazard rate** or **failure rate,** $r(t)$. Life values such as MTTF described above are used to define non-repairable items.

Repairable Items

For repairable items, reliability is the probability that failure will not occur in the time period of interest; or when more than one failure can occur, reliability can be expressed as the **failure rate,** λ, or the **rate of occurrence of failures (ROCOF)**. In the case of repairable items, reliability can be characterized by MTBF described above, but only under the condition of constant failure rate.

There is also the concern for availability, *A(t)*, of repairable items since repair takes time. Availability, *A(t)*, is affected by the rate of occurrence of failures (failure rate, λ) or MTBF plus maintenance time; where maintenance can be corrective (repair) or preventative (to reduce the likelihood of failure). **Availability, *A(t)***, is the probability that an item is in an operable state at any time

$$Availability\ A(t) = \frac{MTBF}{MTBF + MTTR}$$

Some systems are considered both repairable and non-repairable, such as a missile. It is repairable while under test on the ground; but becomes a non-repairable system when fired.

> **NOTE: Failure rate,** λ, is applied loosely to non-repairable items. What is really meant in a repairable system, which contains a part, is that the part will contribute to the overall system failure rate by the stated part failure rate. The part being non-repairable cannot have a failure rate.

Failure Patterns (Non-repairable Items)

There are three patterns of failures for non-repairable items, which can change with time. The failure rate (hazard rate) may be decreasing, increasing or constant.

1. **Decreasing Failure Rate (Non-repairable Items)**

   A decreasing failure rate (DFR) can be caused by an item, which becomes less likely to fail as the survival time increases. This is demonstrated by electronic equipment during their early life or the burn-in period. This is demonstrated by the first half of the traditional bath tub curve for electronic components or equipment where failure rate is decreasing during the early life period.

2. **Constant Failure Rate (Non-repairable Items)**

   A constant failure rate (CFR) can be caused by the application of loads at a constant average rate in excess of the design specifications or strength. These are typically externally induced failures.

3. **Increasing Failure Rate (Non-repairable Items)**

   An increasing failure rate (IFR) can be caused by material fatigue or by strength deterioration due to cyclic loading. Its failure mode does not accrue for a finite time, then exhibits an increasing probability of occurrence.

This failure pattern is also demonstrated by electronic equipment that has aged beyond its useful life (right hand side of the bath tub curve) and the failure rate is rapidly increasing with time.

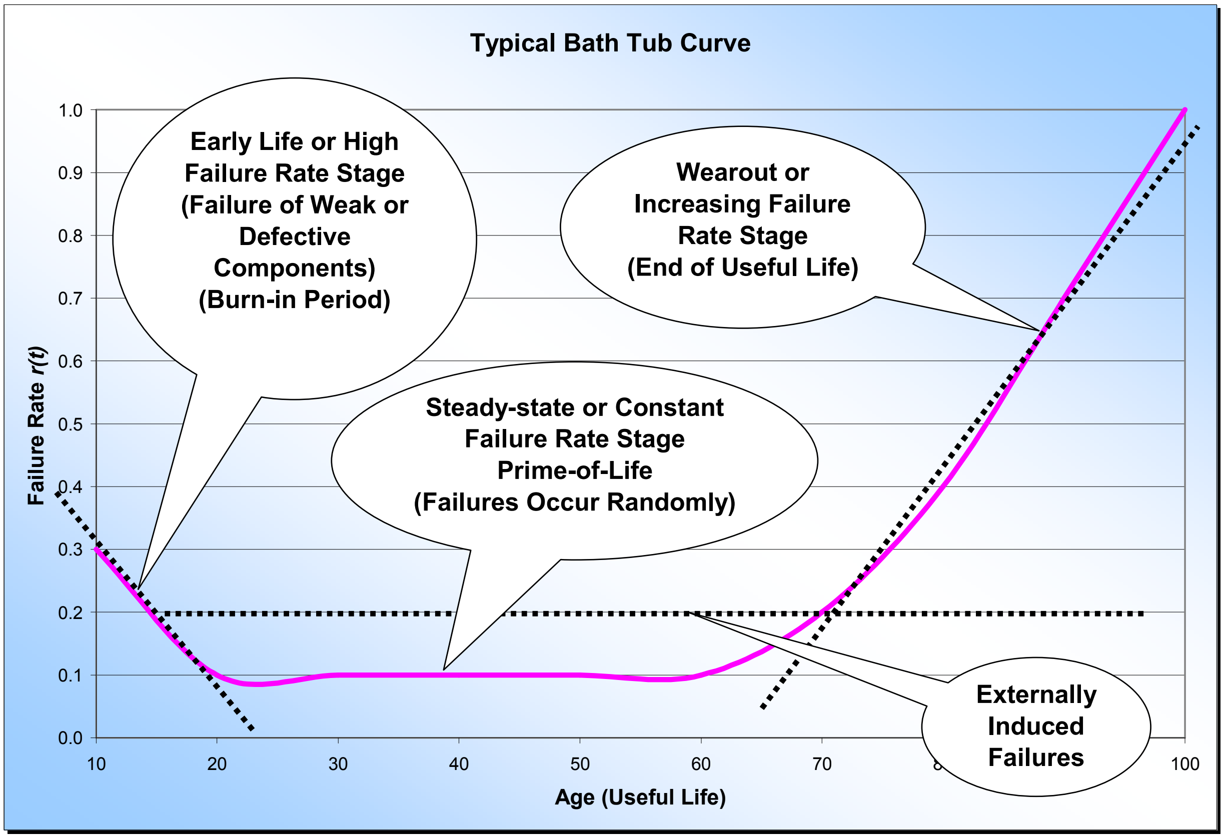

Figure 1 Typical Bath Tub Curve

Failure Patterns (Repairable Items)

There are three patterns of failures for repairable items, which can change with time. The failure rate (hazard rate) may be decreasing, increasing or constant.

1. **Decreasing Failure Rate (Repairable Items)**

   An item whose reliability is improved by progressive repair and / or burn-in can cause a decreasing failure rate (DFR) pattern.

2. **Constant Failure Rate (Repairable Items)**

   A constant failure rate (CFR) is indicative of externally induced failures as in the constant failure rate of non-repairable items. This is typical of complex systems subject to repair and overhaul.

3. **Increasing Failure Rate (Repairable Items)**

This increasing failure rate (IFR) pattern is demonstrated by repairable equipment when wear out modes begin to predominate or electronic equipment that has aged beyond its useful life (right hand side of the bath tub curve) and the failure rate is increasing with time.

Redundancy

Redundancy is briefly defined as the existence of two or more means, not necessarily identical, for accomplishing a given single function. There are different types of redundancy.

- **Active Redundancy** – Has all items operating simultaneously in parallel. All items are working and in use at the same time, even though only one item is required for the function. There is no change in the failure rate of the surviving item after the failure of a companion item.
  - **Pure Parallel** – No Change in the failure rate of surviving items after failure of a companion item.
  - **Shared Parallel** – Failure rate of remaining items change after failure of a companion item.
- **Standby Redundancy** – Has alternate items activated upon failure of the first item. Only one item is operating at a time to accomplish the function. One item's failure rate affects the failure characteristics of others as they are now more susceptible to failure beause they are now under load.
  - **Hot Standby** – Same as Active Standby or Active Redundancy.
  - **Cold Standby (Passive)** – Normally not operating. Do not fail when they are on cold standby. Failure of an item forces standby item to start operating.
  - **Warm Standby** – Normally active or operational, but not under load. Failure rate will be less due to lower stress.
- **R-out-of-n Systems** – Redundant system consisting of n items in which r of the n items must function for the system to function (voting decision).

